

Starting with confidence >>

Mathematics

The following is a list of activities you may like to do during your summer holidays, in preparation for studying A Level Mathematics.

The problems given below illustrate some of the numerical, algebraic and graphical techniques that you will have met at GCSE, which will be developed over the first few weeks of the A Level course.

You should work through the material and ensure you are confident with the techniques needed. Fully worked solutions are provided on the pages that follow.

No calculators allowed! Please show full workings in your answer

1. Solve for x

- a) $2x - 1 = 5$
- b) $7x + 4 = 5x - 2$
- c) $2(4x + 1) = 5x + 8$
- d) $2(1 - x) = 3 + x$

2.

a) Factorise

$$x^2 - 3x$$

b) Hence solve

$$x^2 - 3x = 0$$

3.

a) Factorise

$$x^2 + 3x - 10$$

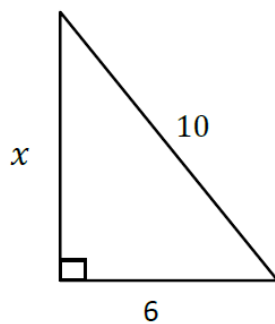
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4. Factorise the following

a) $5x^2 - 2x - 3$

b) $3x^2 + 2x - 1$

5. Use Pythagoras' Theorem to find the value of x



6. Sketch the following graphs, indicating any points of intersection with the axes

a) $y = 2x$

b) $y = 3x - 6$

c) $y = x^2$

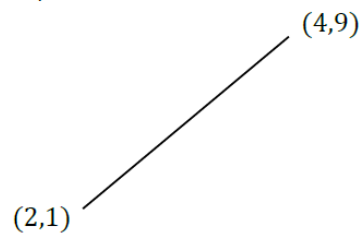
d) $y = x^2 + 4$

e) $y = 2 - x$

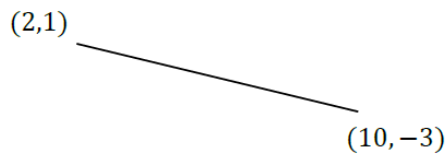
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7. Find the gradients of the lines shown

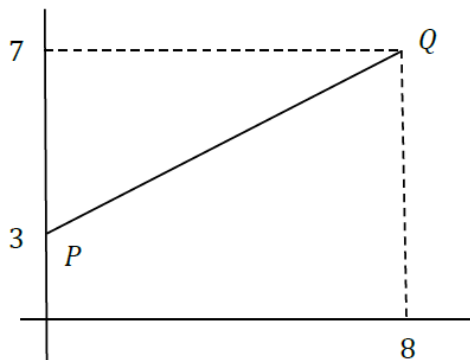
a)



b)



8.



- Find the gradient of the line PQ
- Hence, find the equation of the line PQ

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9. Solve the simultaneous equations for x and y

a)

$$3x + 4y = 7$$

$$x - 2y = 9$$

b)

$$2x + 5y = 18$$

$$x + 3y = 11$$

10. Evaluate

a) $1\frac{1}{2} + 2\frac{1}{4}$

b) $\frac{3}{8} + \frac{1}{2}$

c) $\frac{4}{5} \times \frac{2}{3}$

d) $4 - \frac{2}{3}$

e) $\frac{7}{12} - \frac{1}{3}$

f) $\frac{5}{6} - \frac{1}{4}$

11. Simplify

a) $\sqrt{2} \times \sqrt{2}$

b) $8^{\frac{1}{3}}$

c) $(\sqrt{6})^3$

d) $(5 + \sqrt{2})(2 - \sqrt{2})$ giving
your answer in the form
 $a + b\sqrt{2}$

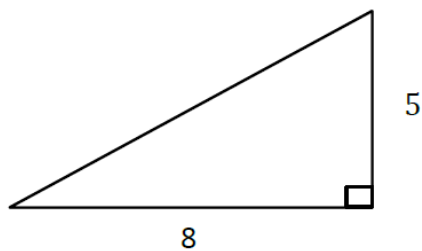
12. Expand and simplify

a) $(2x - 3)(x + 1)$

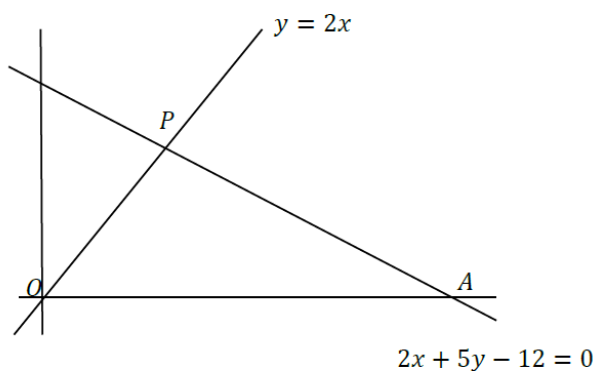
b) $(a + b)^2$

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13. Find the area of the triangle shown



14.



- Calculate the coordinates of point A
- Calculate the coordinates of point P
- Hence find the area of the triangle OAP

15.

- Factorise $x^2 + x - 6$
- Sketch $y = x^2 + x - 6$
- Find the range of values of x for which $x^2 + x - 6 > 0$

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Answers

1.

a)

$$\begin{aligned}2x - 1 &= 5 \\ \Rightarrow 2x &= 5 + 1 \\ \Rightarrow 2x &= 6 \\ \Rightarrow x &= 3\end{aligned}$$

b)

$$\begin{aligned}7x + 4 &= 5x - 2 \\ \Rightarrow 7x - 5x &= -4 - 2 \\ \Rightarrow 2x &= -6 \\ \Rightarrow x &= 3\end{aligned}$$

c)

$$\begin{aligned}2(4x + 1) &= 5x + 8 \\ \Rightarrow 8x + 2 &= 5x + 8 \\ \Rightarrow 8x - 5x &= 8 - 2 \\ \Rightarrow 3x &= 6 \\ \Rightarrow x &= 2\end{aligned}$$

d)

$$\begin{aligned}2(x - 1) &= 3 + x \\ \Rightarrow 2 - 2x &= 3 + x \\ \Rightarrow -2x - x &= 3 - 2 \\ \Rightarrow -3x &= 1 \\ \Rightarrow x &= -\frac{1}{3}\end{aligned}$$

2.

a)

$$x^2 - 3x = x(x - 3)$$

b)

$$\begin{aligned}x^2 - 3x &= 0 \\ \Rightarrow x(x - 3) &= 0 \\ \Rightarrow x = 0 \text{ and } x - 3 &= 0 \\ \Rightarrow x = 0 \text{ and } x &= 3\end{aligned}$$

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3.

a)

$$x^2 + 3x - 10 = (x + 5)(x - 2)$$

b)

$$x^2 + 3x - 10 = 0$$

$$\Rightarrow (x + 5)(x - 2) = 0$$

$$\Rightarrow x + 5 = 0 \text{ and } x - 2 = 0$$

$$\Rightarrow x = -5 \text{ and } x = 2$$

4.

a)

$$5x^2 - 2x - 3 = (5x + 3)(x - 1)$$

b)

$$3x^2 + 2x - 1 = (3x - 1)(x + 1)$$

5.

$$10^2 = x^2 + 6^2$$

$$\Rightarrow x^2 = 10^2 - 6^2$$

$$\Rightarrow x^2 = 100 - 36$$

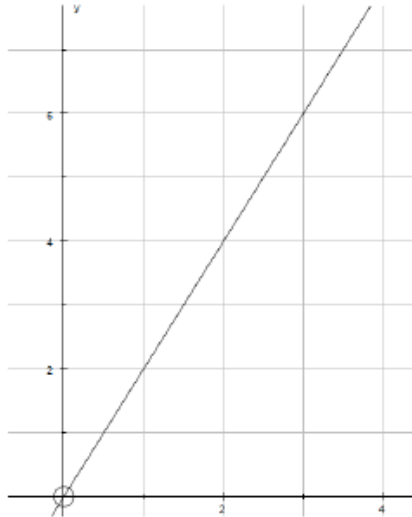
$$\Rightarrow x^2 = 64$$

$$\Rightarrow x = \sqrt{64}$$

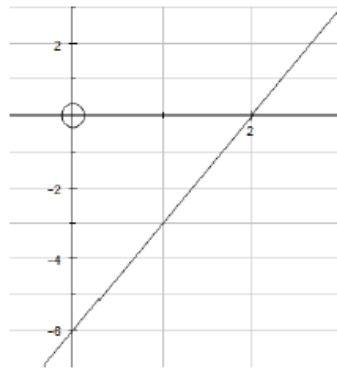
$$\Rightarrow x = 8$$

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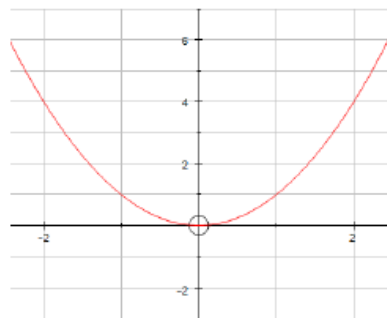
6.
a)



b)

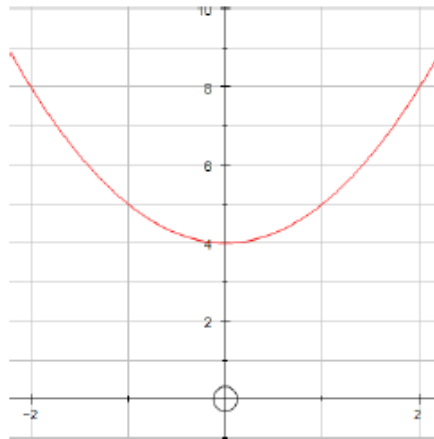


c)

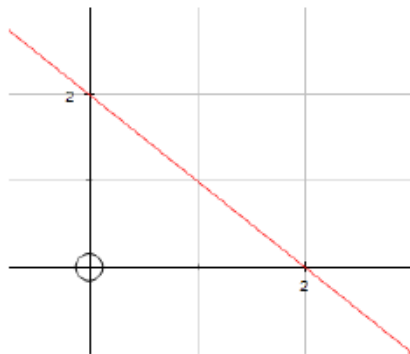


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d)



e)



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7.

a)

$$\text{gradient} = m = \frac{\text{change in } y}{\text{change in } x}$$

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{9 - 1}{4 - 2}$$

$$m = \frac{8}{2}$$

$$m = 4$$

b)

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{-3 - 1}{10 - 2}$$

$$m = \frac{-4}{8}$$

$$m = -\frac{1}{2}$$

8.

a)

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{7 - 3}{8 - 0}$$

$$m = \frac{4}{8}$$

$$m = \frac{1}{2}$$

b)

$$y = mx + c$$

$$y = \frac{1}{2}x + c$$

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9.

a)

$$\begin{aligned}3x + 4y &= 7 \\ x - 2y &= 9\end{aligned}$$

$$(2) \times 2 \Rightarrow 2x - 4y = 18$$

$$(1) + (3) \Rightarrow 3x + 4y + 2x - 4y = 7 + 18$$

$$\Rightarrow 5x = 25$$

$$\Rightarrow x = 5$$

$$\Rightarrow 5 - 2y = 9$$

$$\Rightarrow 2y = 5 - 9$$

$$\Rightarrow 2y = -4$$

$$\Rightarrow y = -2$$

b)

$$\begin{aligned}2x + 5y &= 18 \\ x + 3y &= 11\end{aligned}$$

$$(2) \times 2 \Rightarrow 2x + 6y = 22$$

$$(3) - (1) \Rightarrow 2x - 2x + 6y - 5y = 22 - 18$$

$$\Rightarrow y = 4$$

$$\Rightarrow x + 3(4) = 11$$

$$\Rightarrow x + 12 = 11$$

$$\Rightarrow x = 11 - 12$$

$$\Rightarrow x = -1$$

10.

a)

$$\begin{aligned}1\frac{1}{2} + 2\frac{1}{4} &= \frac{3}{2} + \frac{9}{4} \\ &= \frac{6}{4} + \frac{9}{4} \\ &= \frac{15}{4} \\ &= 3\frac{1}{4}\end{aligned}$$

b)

$$\begin{aligned}\frac{3}{8} + \frac{1}{2} &= \frac{3}{8} + \frac{4}{8} \\ &= \frac{7}{8}\end{aligned}$$

Starting with confidence >>

c)

$$\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} \\ = \frac{8}{15}$$

d)

$$4 - \frac{2}{3} = \frac{12}{3} - \frac{2}{3} \\ = \frac{10}{3} \\ = 3\frac{1}{3}$$

e)

$$\frac{7}{12} - \frac{1}{3} = \frac{7}{12} - \frac{4}{12} \\ = \frac{3}{12} \\ = \frac{1}{4}$$

f)

$$\frac{5}{6} - \frac{1}{4} = \frac{20}{24} - \frac{6}{24} \\ = \frac{14}{24} \\ = \frac{7}{12}$$

11.

a)

$$\sqrt{2} \times \sqrt{2} = (\sqrt{2})^2 \\ = 2$$

b)

$$8^{\frac{1}{3}} = \sqrt[3]{8} \\ = 2$$

c)

$$(\sqrt{6})^3 = (6^{\frac{1}{2}})^3 \\ = 6^{\frac{3}{2}}$$

d)

$$(5 + \sqrt{2})(2 - \sqrt{2}) = 10 - 5\sqrt{2} + 2\sqrt{2} - 2 \\ = 8 - 3\sqrt{2}$$

12.

a)

$$(2x - 3)(x + 1) = 2x^2 - 3x + 2x - 3 \\ = 2x^2 - x - 3$$

b)

$$(a + b)^2 = (a + b)(a + b) \\ = a^2 + 2ab + b^2$$

Starting with confidence >>

13.

$$\begin{aligned} \text{Area} &= \frac{\text{base} \times \text{height}}{2} \\ &= \frac{8 \times 5}{2} \\ &= \frac{40}{2} \\ &= 20 \end{aligned}$$

14.

a)

$$\begin{aligned} \text{At } A: y &= 0 \\ \Rightarrow 2x - 5(0) - 12 &= 0 \\ \Rightarrow 2x &= 12 \\ \Rightarrow x &= 6 \\ \therefore \text{coordinates of } A &= (0,6) \end{aligned}$$

b)

$$\begin{aligned} \text{At } P: y &= 2x \text{ and } 2x + 5y - 12 = 0 \\ \Rightarrow 2x + 5(2x) - 12 &= 0 \\ \Rightarrow 2x + 10x - 12 &= 0 \\ \Rightarrow 12x &= 12 \\ \Rightarrow x &= 1 \end{aligned}$$

$$\begin{aligned} \Rightarrow y &= 2 \\ \therefore \text{coordinates of } P &= (1,2) \end{aligned}$$

c)

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times OA \times \text{height} \\ &= \frac{1}{2} \times 6 \times 2 \\ &= 6 \end{aligned}$$

15.

$$\begin{aligned} \text{length} &= \sqrt{(8-5)^2 + (2-(-2))^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$