

Expanding Polynomials (Answers)

Expanding (Multiplying) Brackets

Expand and simplify:

1. $5(2x - 7)$

$$10x - 35$$

2. $8x(2x + 3)$

$$16x^2 + 24x$$

3. $7a(3a + 2b - 4)$

$$21a^2 + 14ab - 28a$$

4. $5(2x + 1) + 3(x + 4)$

$$10x + 5 + 3x + 12 = 13x + 17$$

5. $8y(y - 4) - 2y(3 - y)$

$$8y^2 - 32y - 6y + 2y^2 = 10y^2 - 38y$$

Remember that a negative times a negative gives a positive!

6. $(3x + 2)(x + 5)$

$$3x^2 + 15x + 2x + 10 = 3x^2 + 17x + 10$$

7. $(x - 4)(3x - 9)$

$$3x^2 - 9x - 12x + 36 = 3x^2 - 21x + 36$$

8. $(a + b)(b - c)$

$$ab - ac + b^2 - bc$$

Note that it is convention to write expressions in decreasing powers, so this might be written instead as $b^2 + ab - ac - bc$

9. $(3x + 2)^2$

$$(3x + 2)(3x + 2) = 9x^2 + 6x + 6x + 4$$

$$= 9x^2 + 12x + 4$$

A common mistake is to simply square both terms. To prevent this, you must write the whole expression out before expanding.

10. $(x + 8)(2x + y - 4)$

$$2x^2 + xy - 4x + 16x + 8y - 32 = 2x^2 + xy + 12x + 8y - 32$$

11. $(x + 3)(x + 4)(x + 1)$

$$(x^2 + 7x + 12)(x + 1) = x^3 + 8x^2 + 19x + 12$$

12. $(2x - 5)(x - 2)(x + 7)$

$$(2x^2 - 9x + 10)(x + 7) = 2x^3 + 5x^2 - 53x + 70$$

13. $(x + 1)^3$

$$(x + 1)(x + 1)(x + 1) = (x^2 + 2x + 1)(x + 1)$$

$$= x^3 + 3x^2 + 3x + 1$$

14. $(x + 2)^2(x + 5)$

$$(x + 2)(x + 2)(x + 5) = (x^2 + 4x + 4)(x + 5)$$

$$= x^3 + 9x^2 + 24x + 20$$

Factorising

Factorise fully:

1. $12x + 15$

$$3(4x + 5)$$

2. $27x - 18$

$$9(3x - 2)$$

3. $10y^2 + 28y$

$2y(5y + 14)$

4. $14ab + 21a$

$7a(2b + 3)$

5. $32x + 40y - 24$

$8(4x + 5y - 3)$

6. $10x^2y - 15xy^2$

$5xy(2x - 3y)$

7. $12a^3b^2 + 18a^2b^3 - 27ab^4$

$3ab^2(4a^2 + 6ab - 9b^2)$

8. $a(b + c) + 5(b + c)$

$(a + 5)(b + c)$

9. $x(y + 3) + 2(y + 3)$

$(x + 2)(y + 3)$

10. $2r(a - 4) - p(a - 4)$

$(2r - p)(a - 4)$

Factorising Quadratic Expressions (Answers)

Factorising: When $a = 1$

Factorise fully:

1. $x^2 + 7x + 10$

$(x + 2)(x + 5)$

2. $x^2 + 12x + 20$

$(x + 10)(x + 2)$

3. $x^2 + 4x - 21$

$(x + 7)(x - 3)$

4. $x^2 - x - 6$

$(x + 2)(x - 3)$

The coefficient of x in this question is -1.

5. $x^2 - 13x + 30$

$(x - 10)(x - 3)$

6. $x^2 - 10x + 25$

$(x - 5)(x - 5)$

This could also be written as $(x - 5)^2$.

Factorising: The Difference of Two Squares

Factorise fully:

1. $x^2 - 36$

$(x + 6)(x - 6)$

4. $25a^2 - b^2$

$(5a + b)(5a - b)$

2. $a^2 - 81$

$(a + 9)(a - 9)$

5. $9x^2 - 100y^2$

$(3x + 10y)(3x - 10y)$

3. $4x^2 - 9$

$(2x + 3)(2x - 3)$

6. $x^4 - y^2$

$(x^2 + y)(x^2 - y)$

Factorising – When $a \neq 1$

Factorise fully:

1. $2x^2 + 11x + 12$

$2 \times 12 = 24$

$8 \times 3 = 24$ and $8 + 3 = 11$

$$\begin{aligned}
 2x^2 + 11x + 12 &= 2x^2 + 8x + 3x + 12 \\
 &= 2x(x + 4) + 3(x + 4) \\
 &= (x + 4)(2x + 3)
 \end{aligned}$$

2. $3x^2 + 26x + 35$

$3 \times 35 = 105$

$5 \times 21 = 105$ and $5 + 21 = 26$

$$\begin{aligned}
 3x^2 + 26x + 35 &= 3x^2 + 5x + 21x + 35 \\
 &= x(3x + 5) + 7(3x + 5) \\
 &= (x + 7)(3x + 5)
 \end{aligned}$$

3. $4x^2 + 8x - 21$

$4 \times -21 = -84$

$14 \times -6 = -84$ and $14 + -6 = 8$

$$\begin{aligned}
 4x^2 + 8x - 21 &= 4x^2 + 14x - 6x - 21 \\
 &= 2x(2x + 7) - 3(2x + 7) \\
 &= (2x + 7)(2x - 3)
 \end{aligned}$$

4. $3x^2 - 19x + 20$

$3 \times 20 = 60$

$-4 \times -15 = 60$ and $-4 + -15 = -19$

$$\begin{aligned}
 3x^2 - 19x + 20 &= 3x^2 - 4x - 15x + 20 \\
 &= x(3x - 4) - 5(3x - 4) \\
 &= (3x - 4)(x - 5)
 \end{aligned}$$

Notice that the common factor for the second pair of expressions needed to be -5 so that the expressions inside the brackets matched.

Completing the Square

Write each equation in completed square form, and then find the coordinates of the turning point.

1. $y = x^2 + 8x + 23$

$$y = (x + 4)^2 + 7$$

$$(-4, 7)$$

2. $y = x^2 - 6x + 1$

$$y = (x - 3)^2 - 8$$

$$(3, -8)$$

3. $y = x^2 + 4x - 6$

$$y = (x + 2)^2 - 10$$

$$(-2, -10)$$

4. $y = x^2 + 3x + 9$

$$y = (x + 1.5)^2 + 6.75 \text{ (or fractional equivalent)}$$

$$(-1.5, 6.75)$$

5. $y = x^2 - 5x - 8$

$$y = (x - 2.5)^2 - 14.25 \text{ (or fractional equivalent)}$$

$$(2.5, -14.25)$$

6. $y = 2x^2 + 12x + 7$

$$y = 2(x^2 + 6x) + 7$$

$$y = 2((x + 3)^2 - 9) + 7$$

$$y = 2(x + 3)^2 - 11$$

$$(-3, -11)$$

7. $y = 3x^2 + 12x + 2$

$$y = 3(x^2 + 4x) + 2$$

$$y = 3((x + 2)^2 - 4) + 2$$

$$y = 3(x + 2)^2 - 10$$

$$(-2, -10)$$

8. $y = 2x^2 + 6x + 23$

$$y = 2(x^2 + 3x) + 23$$

$$y = 2((x + 1.5)^2 - 2.25) + 23$$

$$y = 2(x + 1.5)^2 + 18.5$$

$$(-1.5, 18.5)$$

Linear Equations and Inequalities (Answers)

1. Solve the following equations:

a. $8(2x + 3) = 24$

$$2x + 3 = 3$$

$$2x = 0$$

$$x = 0$$

d. $4(2x - 5) = 3(x + 2)$

$$8x - 20 = 3x + 6$$

$$5x = 26$$

$$x = \frac{26}{5}$$

b. $\frac{3x - 4}{2} = 5$

$$3x - 4 = 10$$

$$3x = 14$$

$$x = \frac{14}{3}$$

e. $\frac{5x - 7}{x} = 9$

$$5x - 7 = 9x$$

$$-7 = 4x$$

$$x = -\frac{7}{4}$$

c. $2\left(\frac{3(x + 1)}{5}\right) = 6$

$$\frac{3(x + 1)}{5} = 3$$

$$3x + 3 = 15$$

$$3x = 12$$

$$x = 4$$

f. $8 - \frac{3x}{2 + x} = 10$

$$-\frac{3x}{2 + x} = 2$$

$$-3x = 4 + 2x \text{ (or equivalent)}$$

$$-5x = 4$$

$$x = -\frac{4}{5}$$

2. Solve the following inequalities:

a. $8x + 3 > 2(x + 5)$

$$8x + 3 > 2x + 10$$

$$6x > 7$$

$$x > \frac{7}{6}$$

b. $\frac{2x - 1}{7} \leq 3$

$$2x - 1 \leq 21$$

$$2x \leq 22$$

$$x \leq 11$$

Linear Equations and Inequalities (Answers)

c. $7 \leq 4x + 5 < 19$

$$2 \leq 4x < 14$$

$$\frac{1}{2} \leq x < \frac{7}{2}$$

d. $5(3 - 2x) \geq 1$

$$15 - 10x \geq 1$$

$$-10x \geq -14$$

$$x \leq \frac{7}{5}$$

3. Find the set of solutions which satisfies the following inequalities:

$$8x \geq 5 - x \text{ and } -4 < 3x + 1 \leq 10$$

Solving the first:

$$9x \geq 5$$

$$x \geq \frac{5}{9}$$

Solving the second:

$$-5 < 3x \leq 9$$

$$-\frac{5}{3} < x \leq 3$$

The set of solutions which satisfies both is $\frac{5}{9} \leq x \leq 3$