Graphs (Answers)

Equations of Straight-Line Graphs / Parallel and Perpendicular Lines

1. Complete the table:

Equation	Gradient	y-intercept
$y = 3x + 7$	**3**	**7**
$y = 2 - x$	**-1**	**2**
$2y = x + 5$	$\frac{1}{2}$ **or 0.5**	$\frac{5}{2}$ **or 2.5**
$3y + 2x = -1$	$-\frac{2}{3}$	$-\frac{1}{3}$
$y = -2x$	-2	0
$y = \frac{3}{4}x - 1$	$\frac{3}{4}$	-1

2. Find the equation of the line passing through the points with coordinates (2, 3) and (4, 1).

$m = \frac{1 - 3}{4 - 2} = -1$

$y = -x + c$

$3 = -2 + c$

$c = 5$

$y = -x + 5$

3. A line whose gradient is $\frac{1}{3}$ passes through the point (-6, 9). Work out the equation of this line, giving your answer in the form $ay + bx = c$, where a, b and c are integers.

$y = \frac{1}{3}x + c$

$9 = \frac{1}{3} \times -6 + c$

$9 = -2 + c$

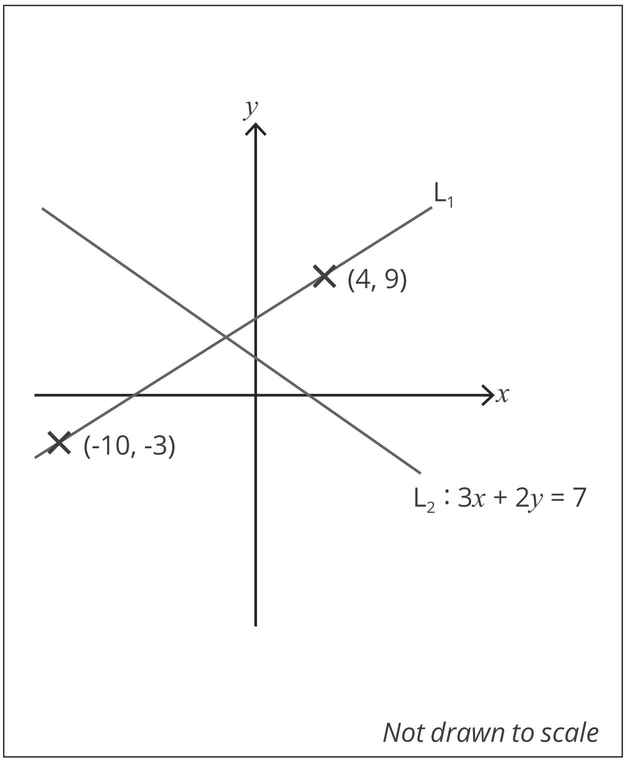
$c = 11$

$y = \frac{1}{3}x + 11$

$3y = x + 33$

$3y - x = 33$

4. The diagram shows two straight lines. Are the lines perpendicular? Justify your answer.



Let m_1 be the gradient of L_1 and m_2 be the gradient of L_2.

$m_1 = \frac{9 - -3}{4 - -10}$

$= \frac{12}{14}$

$= \frac{6}{7}$

Rearrange the equation for L_2 to make y the subject.

$y = -\frac{3}{2}x + \frac{7}{2}$

$m_2 = -\frac{3}{2}$

$\frac{6}{7} \times -\frac{3}{2} \neq -1$ **so the lines are not perpendicular.**

5. Does the line with equation $2x + 5y = -1$ pass through the point with coordinates (2, -1)?

Substitute $x = 2$ and $y = -1$ into the expression $2x + 5y$.

$2 \times 2 + 5 \times (-1) = -1$

Since this is equal to -1, as in the original equation, the line must pass through this point.

Quadratic Graphs

1. Consider the curve with equation $y = x^2 + 4x - 5$.

a. Find the coordinates of the point where this curve intersects the y-axis.

$y = 0^2 + 4 \times 0 - 5 = -5$

(0, -5)

b. Find the coordinates of the points where this curve intersects the x-axis.

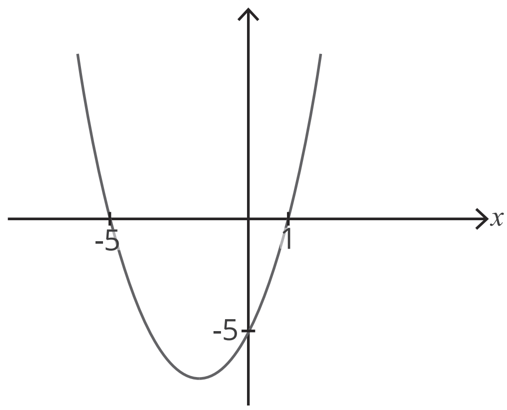
$0 = x^2 + 4x - 5$

$0 = (x - 1)(x + 5)$

$x = 1, x = -5$

(1, 0) and (-5, 0)

c. Hence, sketch the graph of $y = x^2 + 4x - 5$, clearly indicating any points of intersection with the axes.



2. Consider the curve with equation $y = x^2 + 8x - 1$.

a. Find the coordinates of the turning point of this curve.

$\mathbf{x^2 + 8x - 1 = (x + 4)^2 - 4^2 - 1}$

$\mathbf{= (x + 4)^2 - 17}$

The coordinates of the turning point are (-4, -17)

b. State whether the turning point is a maximum or minimum. Justify your answer.

This must be a minimum since the coefficient of x^2 is positive. It is u-shaped and so the turning point must be the lowest point of the curve.

3. Sketch the graph of $y = x^2 + 4x - 21$, clearly indicating any points of intersection with the axes and the location of the turning point of the curve.

$\mathbf{y = 0^2 + 4 \times 0 - 21 = -21}$

$\mathbf{0 = x^2 + 4x - 21}$

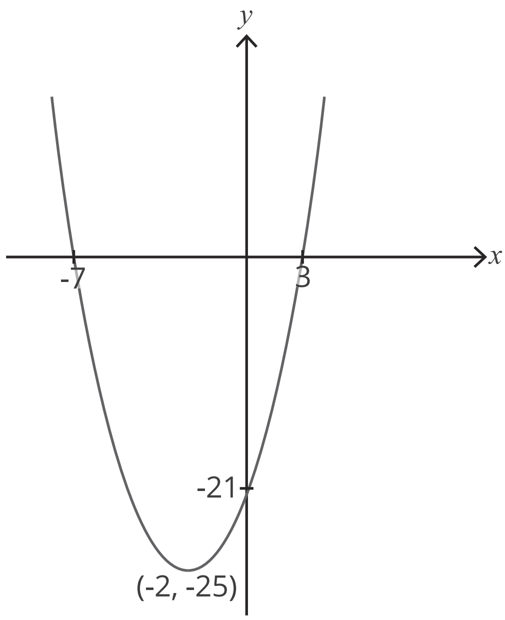
$\mathbf{0 = (x + 7)(x - 3)}$

$\mathbf{x = -7, x = 3}$

$\mathbf{x^2 + 4x - 21 = (x + 2)^2 - 2^2 - 21}$

$\mathbf{= (x + 2)^2 - 25}$

The turning point has coordinates (-2, -25)



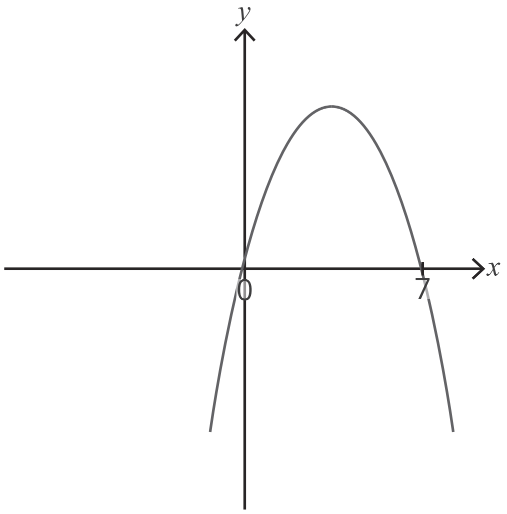
4. Sketch the graph of $y = -x^2 + 7x$, clearly indicating any points of intersection with the axes.

$y = -0^2 + 7 \times 0 = 0$

$0 = -x^2 + 7x$

$0 = -x(x - 7)$

$x = 0$ or $x = 7$



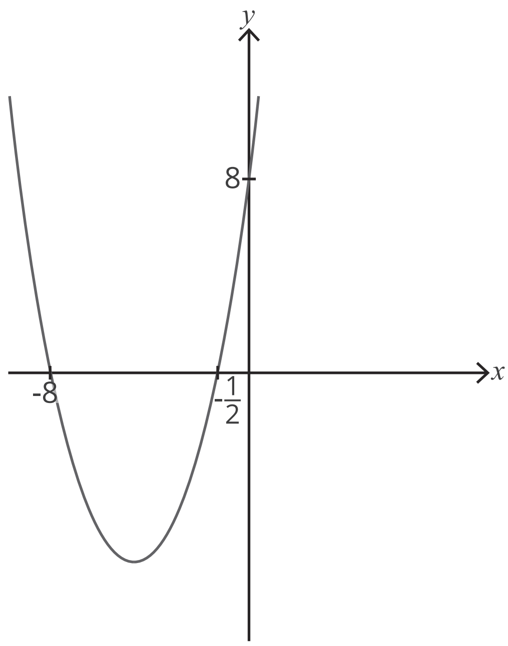
5. Sketch the graph of $y = 2x^2 + 17x + 8$, clearly indicating any points of intersection with the axes.

$y = 2 \times 0^2 + 17 \times 0 + 8 = 8$

$0 = 2x^2 + 17x + 8$

$0 = (2x + 1)(x + 8)$

$x = -\frac{1}{2}$, $x = -8$



Quadratic Equations and Inequalities (Answers)

1. Solve by factorising:

   a. $x^2 - 11x + 28 = 0$

   $(x - 7)(x - 4) = 0$

   $x = 7, x = 4$

   b. $3x^2 - 16x - 12 = 0$

   $(3x + 2)(x - 6) = 0$

   $x = -\frac{2}{3}, x = 6$

   c. $y^2 + 4y - 35 = 2y$

   $y^2 + 2y - 35 = 0$

   $(y - 5)(y + 7) = 0$

   $y = 5, y = -7$

   d. $6n^2 - 6n = -n^2$

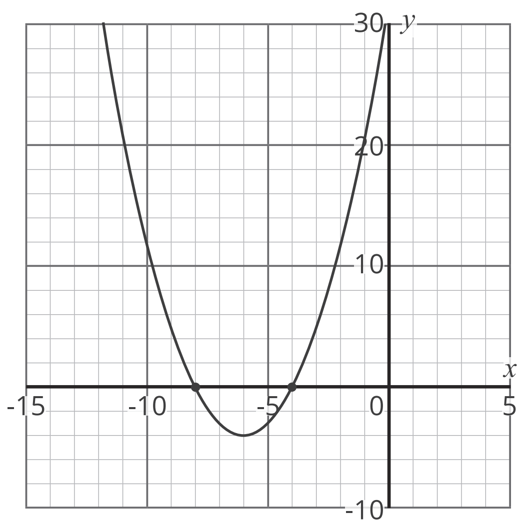
   $7n^2 - 6n = 0$

   $n(7n - 6) = 0$

   $n = 0, n = \frac{6}{7}$

2. Solve the inequalities by sketching the graph:

   a. $x^2 + 12x + 32 \geq 0$



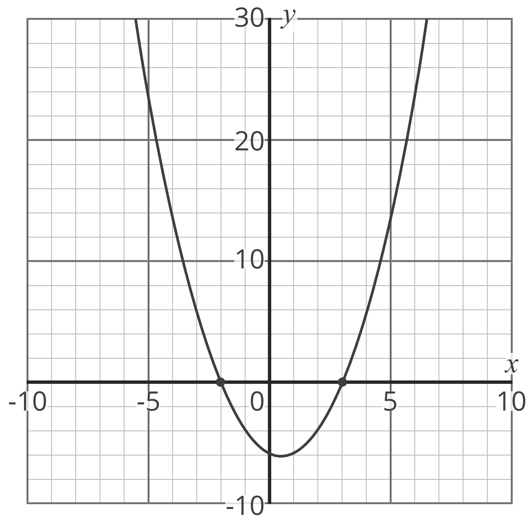
$(x + 8)(x + 4) = 0$

$x = -8, x = -4$

The graph of $y = x^2 + 12x + 32$ is shown as:

$x \leq -8, x \geq -4$

b. $x^2 - 2x - 8 < -x - 2$



$x^2 - x - 6 < 0$

$(x - 3)(x + 2) = 0$

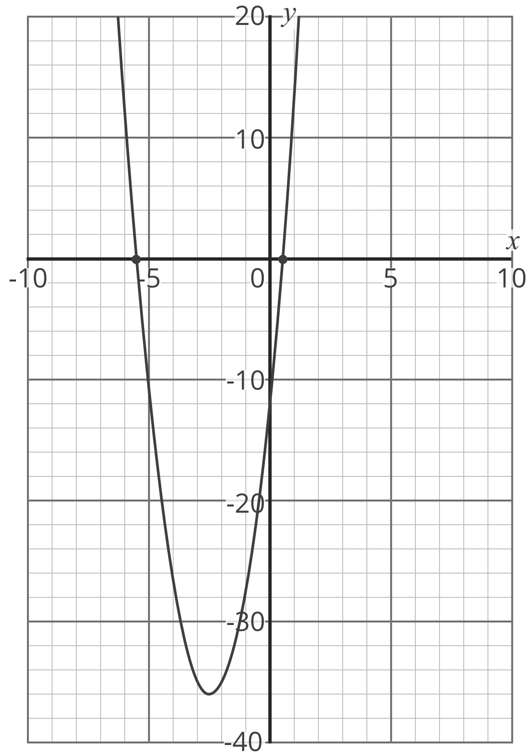
$x = 3, x = -2$

The graph of $y = x^2 - x - 6$

is shown as:

$-2 < x < 3$

c. $4x^2 + 20x - 11 \leq 0$



$(2x - 1)(2x + 11) = 0$

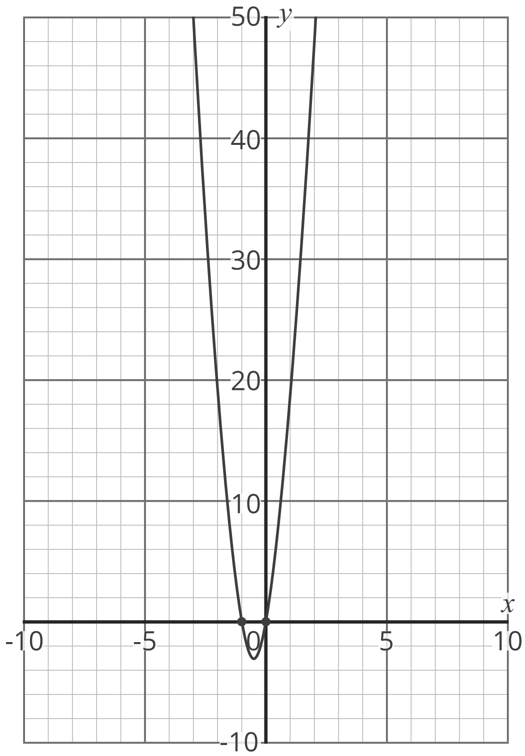
$x = \frac{1}{2}, x = -\frac{11}{2}$

The graph of $y = 4x^2 + 20x - 11$

is shown as:

$-\frac{11}{2} \leq x \leq \frac{1}{2}$

d. $8x^2 + 9x > 2x$



$8x^2 + 7x > 0$

$x(8x + 7) = 0$

$x = 0, x = -\frac{7}{8}$

The graph of $y = 8x^2 + 7x$

is shown as:

$x < -\frac{7}{8}, x > 0$

3. Solve by completing the square, writing surds in their simplest form:

a. $x^2 + 8x + 3 = 0$

$(x + 4)^2 - 13 = 0$

$(x + 4)^2 = 13$

$x + 4 = \pm\sqrt{13}$

$x = -4 \pm \sqrt{13}$

b. $x^2 + 3x - 7 = 0$

$(x + \frac{3}{2})^2 - \frac{37}{4} = 0$

$(x + \frac{3}{2})^2 = \frac{37}{4}$

$x + \frac{3}{2} = \pm\sqrt{\frac{37}{4}}$

$x = -\frac{3}{2} \pm \sqrt{\frac{37}{4}}$

$x = -\frac{3}{2} \pm \frac{\sqrt{37}}{2}$

c. $2x^2 + 12x + 1 = 0$

$2(x^2 + 6x) + 1 = 0$

$2((x + 3)^2 - 9) + 1 = 0$

$2(x + 3)^2 - 17 = 0$

$2(x + 3)^2 = 17$

$(x + 3)^2 = \frac{17}{2}$

$x + 3 = \pm\sqrt{\frac{17}{2}}$

$x = -3 \pm \sqrt{\frac{17}{2}}$

d. $x^2 + 16x + 3 = -9$

$x^2 + 16x + 12 = 0$

$(x + 8)^2 - 52 = 0$

$(x + 8)^2 = 52$

$x + 8 = \pm\sqrt{52}$

$x = -8 \pm \sqrt{52}$

$x = -8 \pm 2\sqrt{13}$

The Quadratic Formula (Answers)

1. Solve $2x^2 + 5x + 1 = 0$, giving your answers correct to 3 significant figures.

   $$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times 1}}{2 \times 2}$$

   $x = -2.28$, $x = -0.219$

2. Solve $5x^2 + 2x = 19$, giving your answers correct to 3 significant figures.

   $5x^2 + 2x - 19 = 0$

   $$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 5 \times (-19)}}{2 \times 5}$$

   $x = -2.16$, $x = 1.76$

3. Explain why the graph of the equation $y = x^2 + 4x + 9$ does not intersect the x-axis.
   Use the discriminant $\Delta = b^2 - 4ac$

   $\Delta = 4^2 - 4 \times 1 \times 9$

   $\Delta = -20$

   $\Delta < 0$

   Since the discriminant is less than zero, there are no real solutions to the equation $x^2 + 4x + 9 = 0$ and so the graph does not intersect the x-axis.

Simultaneous Equations (Answers)

1. Solve each pair of simultaneous equations:

a. $x + y = 14$
$2x - y = 16$

Add the equations:

$3x = 30$

$x = 10$

$10 + y = 14$

$y = 4$

$x = 10, y = 4$

b. $3x + 2y = -4$
$2x + y = -3$

Multiply the second equation by 2, then subtract the equations:

$4x + 2y = -6$

$x = -2$

$2 \times (-2) + y = -3$

$-4 + y = -3$

$y = 1$

$x = -2, y = 1$

c. $2x + 5y = 26$
$y = x + 1$

Substitute the second equation into the first. You can use elimination, but you will need to rearrange first.

$2x + 5(x + 1) = 26$

$7x + 5 = 26$

$7x = 21$

$x = 3$

$y = 3 + 1$

$y = 4$

$x = 3, y = 4$

d. $x^2 + y^2 = 10$
$y = x + 2$

Substitute the second equation into the first.

$x^2 + (x + 2)^2 = 10$

$2x^2 + 4x + 4 = 10$

$2x^2 + 4x - 6 = 0$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3$ or $x = 1$

$y = -3 + 2$ or $y = 1 + 2$

$y = -1$ or 3

$x = -3$ and $y = -1$, $x = 1$ and $y = 3$

e. $2x^2 = y^2 - 8$

$y - x = 2$

Rearrange and then substitute the second equation into the first.

$y = x + 2$

$2x^2 = (x + 2)^2 - 8$

$2x^2 = x^2 + 4x + 4 - 8$

$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$x = 2$

$y = 2 + 2 = 4$

$x = 2, y = 4$