

Negative and Zero Powers (Answers)

1. a. What is $3^5 \div 3^2$ in index form?

$$3^3$$

d. Evaluate 3^0

$$1$$

b. What is $3^2 \div 3^2$ in index form?

$$3^0$$

e. Evaluate 27.54^0

$$1$$

c. Evaluate $3^2 \div 3^2$

$$1$$

f. Evaluate $2.7523^0 \times 268^1 \times 892^0$

$$268$$

2. Evaluate the following:

a. 5^{-2}

$$\frac{1}{25}$$

b. 8^{-2}

$$\frac{1}{64}$$

c. 3^{-3}

$$\frac{1}{27}$$

d. 2^{-5}

$$\frac{1}{32}$$

3. Write each in index form:

a. $\frac{1}{16}$

$$4^{-2}$$

b. $\frac{1}{49}$

$$7^{-2}$$

c. $\frac{1}{125}$

$$5^{-3}$$

d. $\frac{1}{1000}$

$$10^{-3}$$

4. Evaluate, giving your answers as fractions in their simplest form:

a. $(\frac{3}{5})^{-1}$

$$\frac{5}{3}$$

b. $(\frac{7}{8})^{-2}$

$$\frac{64}{49}$$

c. $(\frac{1}{4})^{-3}$

$$\frac{64}{1} = 64$$

d. $(\frac{2}{3})^{-3}$

$$\frac{27}{8}$$

5. Evaluate, giving your answers as fractions in their simplest form:

a. $(3x)^{-2}$

$$\frac{1}{9x^2}$$

b. $(2x^3)^{-2}$

$$\frac{1}{4x^6}$$

c. $(5x^4)^{-3}$

$$\frac{1}{125x^{12}}$$

d. $(2x^2y^3)^{-4}$

$$\frac{1}{16x^8y^{12}}$$

Fractional Powers (Answers)

1. Evaluate the following:

a. $36^{\frac{1}{2}}$

$$\sqrt{36} = 6$$

b. $1000^{\frac{1}{3}}$

$$\sqrt[3]{1000} = 10$$

c. $64^{\frac{1}{3}}$

$$\sqrt[3]{64} = 4$$

d. $81^{-\frac{1}{2}}$

$$\frac{1}{\sqrt{81}} = \frac{1}{9}$$

2. Evaluate the following:

a. $27^{\frac{2}{3}}$

$$= \sqrt[3]{27^2}$$

$$= 3^2$$

$$= 9$$

b. $8^{\frac{4}{3}}$

$$= \sqrt[3]{8^4}$$

$$= 2^4$$

$$= 16$$

c. $49^{\frac{3}{2}}$

$$= \sqrt{49^3}$$

$$= 7^3$$

$$= 343$$

d. $64^{\frac{2}{3}}$

$$= \sqrt[3]{64^2}$$

$$= 4^2$$

$$= 16$$

3. Express in the form $a^{\frac{m}{n}}$, where m and n are integers.

a. $\sqrt{a^3}$

$$a^{\frac{3}{2}}$$

c. $\frac{1}{\sqrt{a^7}}$

$$a^{-\frac{7}{2}}$$

b. $\sqrt[3]{a^5}$

$$a^{\frac{5}{3}}$$

d. $\sqrt{a} \times \frac{1}{\sqrt{a^5}}$

$$a^{\frac{1}{2}} \times a^{-\frac{5}{2}}$$

$$= a^{\frac{1}{2} + -\frac{5}{2}}$$

$$= a^{-\frac{4}{2}}$$

$$\text{or simplify to } a^{-2}$$

4. Write the following expressions in order, from smallest to largest:

$$25^{\frac{1}{2}},$$

$$8^{\frac{2}{3}},$$

$$27^{\frac{1}{3}},$$

$$\left(\frac{1}{9}\right)^{\frac{3}{2}},$$

$$\left(\frac{1}{12}\right)^{-1},$$

$$(27^{\frac{5}{3}})^0,$$

$$25^{\frac{1}{2}} = 5, \quad 8^{\frac{2}{3}} = 4, \quad 27^{\frac{1}{3}} = 3, \quad \left(\frac{1}{9}\right)^{-\frac{3}{2}} = 27, \quad \left(\frac{1}{12}\right)^{-1} = 12, \quad (27^{\frac{5}{3}})^0 = 1$$

$$(27^{\frac{5}{3}})^0, \quad 27^{\frac{1}{3}}, \quad 8^{\frac{2}{3}}, \quad 25^{\frac{1}{2}}, \quad \left(\frac{1}{12}\right)^{-1}, \quad \left(\frac{1}{9}\right)^{-\frac{3}{2}}$$

5. Write $64^{\frac{2}{3}} \times 2^3$ in the form 2^a , where a is a positive integer.

$$64 = 2^6$$

$$64^{\frac{2}{3}} = (2^6)^{\frac{2}{3}}$$

$$= 2^{\frac{12}{3}} \quad \text{(use the index law for brackets)}$$

$$= 2^4$$

$$64^{\frac{2}{3}} \times 2^3 = 2^4 \times 2^3$$

$$= 2^7 \quad \text{(use the index law for multiplication)}$$

Index Laws (Answers)

1. Simplify each expression. Give your answers in index form.

a. $5^4 \times 5^8$

$$5^{4+8} = 5^{12}$$

b. $m^4 \div m^2$

$$m^{4-2} = m^2$$

c. $(a^3)^2$

$$a^{3 \times 2} = a^6$$

d. $3^5 \times 3$

$$3^{5+1} = 3^6$$

2. Simplify each expression. Give your answers in index form.

a. $3^8 \times 3^{-2}$

$$3^{8+(-2)} = 3^6$$

b. $\frac{h^{-3}}{h^5}$

$$h^{-3-5} = h^{-8}$$

c. $p^{-2} \div p^{-9}$

$$p^{-2-(-9)} = p^7$$

d. $(5^{-3})^{-2}$

$$5^{-3 \times -2} = 5^6$$

3. Simplify each expression. Give your answers in index form.

a. $3a^2 \times 3a^5$

$$9a^7$$

b. $(3x^4)^3$

$$27x^{12}$$

c. $\frac{12x^3}{4x^5}$

$$3x^{-2}$$

d. $a^2b^5 \times a^4b^{-8}$

$$a^6b^{-3}$$

4. Simplify the expression. Give your answer in index form.

$$\begin{aligned} \left(\frac{3a^5 \times 6a^{-7}}{2a^5} \right)^2 &= \left(\frac{18a^{-2}}{2a^5} \right)^2 \\ &= (9a^{-7})^2 \\ &= 81a^{-14} \end{aligned}$$

Simplifying Surds (Answers)

Multiplying and Dividing Surds

1. a. $\sqrt{5} \times \sqrt{7}$

$$\sqrt{35}$$

d. $18\sqrt{20} \div 6\sqrt{5}$

$$3\sqrt{4} = 3 \times 2 = 6$$

g. $(5\sqrt{3})^2$

$$25 \times 3 = 75$$

b. $3\sqrt{2} \times 4\sqrt{5}$

$$12\sqrt{10}$$

e. $5\sqrt{2} \times 3\sqrt{8}$

$$15\sqrt{16}$$

$$= 15 \times 4 = 60$$

h. $(2\sqrt{5})^3$

$$2^3 \times (\sqrt{5})^3$$

$$= 8 \times 5\sqrt{5} = 40\sqrt{5}$$

c. $\sqrt{15} \div \sqrt{3}$

$$\sqrt{5}$$

f. $2\sqrt{3} \times 5$

$$10\sqrt{3}$$

2. A right-angled triangle has a height of $6\sqrt{5}\text{cm}$ and a base of $7\sqrt{3}\text{cm}$. Find its area.

$$6\sqrt{5} \times 7\sqrt{3} \div 2$$

$$= 42\sqrt{15} \div 2$$

$$= 21\sqrt{15}\text{cm}^2$$

Addition and Subtraction of Surds

1. Simplify these surds (remember: the key is to find a square factor).

a. $\sqrt{20}$

$$\sqrt{4} \sqrt{5} = 2\sqrt{5}$$

b. $\sqrt{48}$

$$\sqrt{16} \sqrt{3} = 4\sqrt{3}$$

c. $\sqrt{75}$

$$\sqrt{25} \sqrt{3} = 5\sqrt{3}$$

d. $5\sqrt{8}$

$$5(2\sqrt{2}) = 10\sqrt{2}$$

2. Give your answers in the form $a\sqrt{b}$

a. $\sqrt{2} + \sqrt{18}$

$$\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$$

b. $\sqrt{50} - \sqrt{200}$

$$\sqrt{25} \sqrt{2} - \sqrt{100} \sqrt{2}$$

$$= 5\sqrt{2} - 10\sqrt{2} = -5\sqrt{2}$$

c. $4\sqrt{80} + 3\sqrt{45}$

$$4\sqrt{16} \sqrt{5} + 3\sqrt{9} \sqrt{5}$$

$$= 16\sqrt{5} + 9\sqrt{5} = 25\sqrt{5}$$

d. $2\sqrt{50} + 5\sqrt{32}$

$$2\sqrt{25} \sqrt{2} + 5\sqrt{16} \sqrt{2}$$

$$= 10\sqrt{2} + 20\sqrt{2} = 30\sqrt{2}$$

3. A rectangle has a width of $6\sqrt{75}\text{m}$ and a height of $2\sqrt{12}\text{m}$. What is its perimeter?

$$\sqrt{75} = 5\sqrt{3}$$

$$\sqrt{12} = 2\sqrt{3}$$

$$6\sqrt{75} + 6\sqrt{75} + 2\sqrt{12} + 2\sqrt{12}$$

$$= 12\sqrt{75} + 4\sqrt{12}$$

$$= 60\sqrt{3} + 8\sqrt{3} = 68\sqrt{3}\text{m}$$

Challenge

A right-angled triangle has a base of $2\sqrt{18}\text{cm}$ and a height of $2\sqrt{32}\text{cm}$. Find the perimeter of the triangle.

Using Pythagoras to find the hypotenuse:

$$\sqrt{(2\sqrt{18})^2 + (2\sqrt{32})^2}$$

$$= \sqrt{72 + 128}$$

$$= \sqrt{200} \text{ cm}$$

Add the three sides:

$$2\sqrt{18} + 2\sqrt{32} + \sqrt{200}$$

$$= 6\sqrt{2} + 8\sqrt{2} + 10\sqrt{2} = 24\sqrt{2}\text{cm}$$

Rationalising the Denominator (Answers)

1. Rationalise the denominator of each fraction.

a. $\frac{5}{\sqrt{2}}$

$$\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

c. $\frac{\sqrt{2}}{\sqrt{3}}$

$$\frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

e. $\frac{2-\sqrt{3}}{\sqrt{3}}$

$$\frac{2-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}-3}{3}$$

b. $\frac{4}{\sqrt{3}}$

$$\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

d. $\frac{3}{2\sqrt{5}}$

$$\frac{3}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{10}$$

2. Rationalise the denominator of $\frac{\sqrt{5}}{\sqrt{80}}$. Give your answer as a fraction in its simplest form.

$$\frac{\sqrt{5}}{\sqrt{80}}$$

$$= \frac{\sqrt{5}}{\sqrt{80}} \times \frac{\sqrt{80}}{\sqrt{80}}$$

$$= \frac{\sqrt{400}}{80}$$

$$= \frac{20}{80}$$

$$= \frac{1}{4}$$

Or: $\frac{\sqrt{5}}{\sqrt{80}}$

$$= \frac{\sqrt{5}}{\sqrt{16}\sqrt{5}}$$

$$= \frac{\sqrt{5}}{4\sqrt{5}}$$

$$= \frac{1}{4}$$

3. What is $\frac{2\sqrt{2}}{\sqrt{6}} + \frac{1}{\sqrt{3}}$? Give your answer in its simplest terms.

$$\frac{2\sqrt{2}}{\sqrt{6}}$$

$$= \frac{2\sqrt{2}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{2\sqrt{12}}{6}$$

$$= \frac{\sqrt{12}}{3}$$

$$= \frac{2\sqrt{3}}{3}$$

$$\frac{1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{3}$$

Therefore, $\frac{2\sqrt{2}}{\sqrt{6}} + \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3}$

$$= \frac{3\sqrt{3}}{3}$$

$$= \sqrt{3}$$

4. a. $\frac{5}{\sqrt{2} + 7}$

$$\frac{5}{\sqrt{2} + 7} \times \frac{\sqrt{2} - 7}{\sqrt{2} - 7}$$

$$= \frac{5(\sqrt{2} - 7)}{(\sqrt{2} + 7)(\sqrt{2} - 7)}$$

$$= \frac{35 - 5\sqrt{2}}{47} \text{ or equivalent}$$

b. $\frac{1}{\sqrt{5} - 3}$

$$\frac{1}{\sqrt{5} - 3} \times \frac{\sqrt{5} + 3}{\sqrt{5} + 3}$$

$$= \frac{-\sqrt{5} - 3}{4} \text{ or equivalent}$$

c. $\frac{1 + \sqrt{2}}{\sqrt{3} + 2}$

$$\frac{1 + \sqrt{2}}{\sqrt{3} + 2} \times \frac{\sqrt{3} - 2}{\sqrt{3} - 2}$$

$$= \frac{(1 + \sqrt{2})(\sqrt{3} - 2)}{(\sqrt{3} + 2)(\sqrt{3} - 2)}$$

$$= \frac{\sqrt{3} - 2 + \sqrt{6} - 2\sqrt{2}}{(\sqrt{3} + 2)(\sqrt{3} - 2)}$$

$$= \frac{\sqrt{3} - 2 + \sqrt{6} - 2\sqrt{2}}{3 - 4}$$

$$= 2 + 2\sqrt{2} - \sqrt{3} - \sqrt{6} \text{ or equivalent}$$

Challenge

Amy is laying tiles in her rectangular bathroom. By the time she has finished, she has used 8m^2 worth of tiles. She knows the length of one side of the room is $(\sqrt{5} + 2)\text{m}$ but, unfortunately, she has lost her tape measure. Amy still needs to work out the perimeter of the room. Calculate the perimeter of the room, giving your answer in its simplest form.

$$\text{Missing length} = \frac{8}{\sqrt{5} + 2}$$

$$= \frac{8\sqrt{5} - 16}{1}$$

$$= (8\sqrt{5} - 16)\text{m}$$

$$\text{Perimeter} = 2(8\sqrt{5} - 16) + 2(\sqrt{5} + 2)$$

$$= 16\sqrt{5} - 32 + 2\sqrt{5} + 4$$

$$= (18\sqrt{5} - 28)\text{m}$$